

Name: _____

Date: _____

Note: Rational Numbers – Topic B

Lesson 13 – Converting Between Fractions and Decimals Using Equivalent Fractions

Rational Numbers –

Any number that can be expressed as a fraction.

Terminating Decimals –

a decimal whose digits end at some point.

(Repeating decimals go on forever) ex: $0.3333\ldots \rightarrow 0.\overline{3}$

Rules: Decimals $\rightarrow$ Fractions

1. Put the # over its place value #.
2. Reduce/Simplify
($\div$ top & bottom by the same #).

Fractions $\rightarrow$ Decimals

1. Divide numerator by denominator
2. "Top # goes under"
3. Use long division

Use place value to convert each terminating decimal to a fraction. Write each fraction in its simplest form.

1) $0.16 \leftarrow 100^{\text{ths}}$

$$\frac{16}{100} \div 2 = \frac{8}{50} \div 2 = \boxed{\frac{4}{25}}$$

2) $2.72 \leftarrow 100^{\text{ths}}$

$$2 \frac{72}{100} \div 2 = 2 \frac{36}{50} \div 2 = \boxed{2 \frac{18}{25}}$$

3) $-0.218 \leftarrow 1000^{\text{ths}}$

$$-\frac{218}{1000} \div 2 = \boxed{-\frac{109}{500}} \leftarrow \text{prime}$$

4) $0.0005 \leftarrow 10,000^{\text{ths}}$

$$\frac{5}{10,000} \div 5 = \boxed{\frac{1}{2000}}$$

$$\begin{array}{r} 109 \\ 2 \overline{) 218} \\ \underline{-2} \\ 0 \\ \underline{-0} \\ 18 \end{array}$$

Convert each Fraction to a Decimal.

5) $\frac{3}{20}$

$$\begin{array}{r} 0.15 \\ 20 \overline{) 3.00} \\ \underline{20} \downarrow \\ 100 \\ \underline{100} \\ 0 \end{array}$$

0.15

6) $\frac{7}{5}$

$$\begin{array}{r} 1.4 \\ 5 \overline{) 7.0} \\ \underline{5} \downarrow \\ 20 \\ \underline{20} \\ 0 \end{array}$$

1.4

7) $\frac{3}{16}$

$$\begin{array}{r} 0.1875 \\ 16 \overline{) 3.0000} \\ \underline{16} \downarrow \\ 140 \\ \underline{128} \downarrow \\ 120 \\ \underline{112} \downarrow \\ 80 \end{array}$$

0.1875

8) $-\frac{35}{50}$

$$\begin{array}{r} 0.7 \\ 50 \overline{) 35.0} \\ \underline{35} \\ 0 \end{array}$$

-0.7

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Lesson 13 Practice

Converting Between Fractions & Decimals Using Equivalent Fractions

Use place value to convert each terminating decimal to a fraction. Then rewrite each fraction in its simplest form.

1. -0.4 $-\frac{4 \div 2}{10 \div 2} = \boxed{-\frac{2}{5}}$

2. 3.08 $3\frac{8 \div 4}{100 \div 4} = \boxed{3\frac{2}{25}}$

3. 0.625 $\frac{625 \div 5}{1000 \div 5} = \frac{125 \div 5}{200 \div 5} = \frac{25 \div 5}{40 \div 5} = \boxed{\frac{5}{8}}$

4. 0.012 $\frac{12 \div 2}{1000 \div 2} = \frac{6 \div 2}{500 \div 2} = \boxed{\frac{3}{250}}$

5. Write 2.25 as a fraction. Explain your process.

Write $2\frac{25}{100}$ because 25 is in the hundredths ~~pos~~ position, then divide the numerator and denominator by 25 to get $2\frac{1}{4}$.

Convert each fraction to a decimal.

6. $\frac{4}{5}$ 0.8

7. $-\frac{3}{40}$ -0.075

8. $\frac{20}{8}$ 2.5

9. $\frac{5}{16}$ 0.3125

Converting Rational Numbers to Decimals Using Long Division

Repeating Decimals –

Continue on forever in a specific pattern of digits, use repeating bar $\overline{}$

1. Using the division button on your calculator, explore various quotients of integers 1 through 11.

$$\begin{array}{cccccc} \frac{1}{2} & 0.5 & \frac{1}{3} & 0.\overline{3} & \frac{1}{4} & 0.25 & \frac{1}{5} & 0.2 & \frac{1}{6} & 0.1\overline{6} \\ \frac{1}{7} & 0.142857 & \frac{1}{8} & 0.125 & \frac{1}{9} & 0.\overline{1} & \frac{1}{10} & 0.1 & \frac{1}{11} & 0.0\overline{9} \end{array}$$

2. In the chart below, organize the fractions and their corresponding decimal representation listed in Example 1 according to their type of decimal.

Terminating Decimal	Repeating Decimal
$\frac{1}{2}, \frac{1}{4}, \frac{1}{5},$ $\frac{1}{8}, \frac{1}{10}$	$\frac{1}{3}, \frac{1}{6}, \frac{1}{7},$ $\frac{1}{9}, \frac{1}{11}$

What do these numbers have in common?

Denominators are divisible by 2 and/or 5.

What do these numbers have in common?

Denominators are not divisible by only 2 and/or 5.

3. Calculate the decimal values of the fraction below using long division.

a. $\frac{4}{9}$

$$\begin{array}{r} 0.444 \\ 9 \overline{) 4.000} \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \end{array}$$

$0.\overline{4}$

b. $5\frac{4}{11}$

$$\begin{array}{r} 0.3636 \\ 11 \overline{) 4.000} \\ \underline{33} \\ 70 \\ \underline{66} \\ 40 \\ \underline{33} \\ 70 \\ \underline{66} \end{array}$$

$5.\overline{36}$

c. $-\frac{5}{6}$

$$\begin{array}{r} 0.8333 \\ 6 \overline{) 5.0000} \\ \underline{48} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \end{array}$$

$-0.\overline{83}$

4. Eric and four of his friends are taking a trip across the New York State Thruway. They decide to split the cost of tolls equally. If the total cost of tolls is \$8, how much will each person have to pay?

$\frac{\$8}{5 \text{ people}}$

$$\begin{array}{r} 1.6 \\ 5 \overline{) 8.0} \\ \underline{5} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

$\$1.60$

5. Just before leaving on the trip, two of Eric's friends have a family emergency and cannot go. What is each person's share of the \$8 tolls now?

$\frac{\$8}{3 \text{ people}}$

$$\begin{array}{r} 2.666... \\ 3 \overline{) 8.0} \\ \underline{6} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \end{array}$$

\$ rounds to nearest cent (100th)

$\$2.67$

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Lesson 14 Practice

Converting Rational Numbers to Decimals Using Long Division

1. Calculate the decimal values of the fraction below using long division.

a. $\frac{2}{3}$

$$\begin{array}{r} 0.666 \\ 3 \overline{) 2.0} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \end{array}$$

$0.\overline{6}$

b. $-\frac{1}{6}$

$$\begin{array}{r} 0.166 \\ 6 \overline{) 1.0} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \end{array}$$

$-0.\overline{16}$

c. $3\frac{7}{9}$

$$\begin{array}{r} 0.777 \\ 9 \overline{) 7.0} \\ \underline{63} \\ 70 \\ \underline{63} \\ 70 \end{array}$$

$3.\overline{7}$

d. $\frac{8}{11}$

$$\begin{array}{r} 0.7272 \\ 11 \overline{) 8.0} \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \end{array}$$

$0.\overline{72}$

e. $\frac{3}{6}$

$$\begin{array}{r} 0.5 \\ 6 \overline{) 3.0} \\ \underline{30} \\ 0 \end{array}$$

0.5

f. $\frac{5}{37}$

$$\begin{array}{r} 0.1351 \\ 37 \overline{) 5.0} \\ \underline{37} \\ 130 \\ \underline{111} \\ 190 \\ \underline{185} \\ 50 \end{array}$$

$0.\overline{135}$

2. One of these decimal representations is not like the others in question 1 above. Why?

$\frac{3}{6} = \frac{1}{2}$ doesn't repeat
it terminates.

Addition and Subtraction of Rational Numbers

RULES: Adding and Subtracting Fractions

1. Find a common denominator
2. multiply numerators and denominators to get equivalent fractions
3. add/subtract whole #'s (borrow if necessary)
4. add/subtract numerators, leave denominators the same.
5. reduce / simplify if necessary

Find the following sums or differences in simplest form.

$$1. \frac{9}{10} + \frac{3}{10} = \frac{12}{10}$$

$$1 \frac{2 \div 2}{10 \div 2} = 1 \frac{1}{5}$$

$$2. \frac{11}{12} + \left(-\frac{5}{12}\right) = \frac{6 \div 2}{12 \div 2}$$

$$\frac{1}{2}$$

$$3. 3\frac{5}{8} - 2\frac{7}{8}$$

$$2\frac{8}{8} + \frac{5}{8}$$

$$2\frac{13}{8} - 2\frac{7}{8}$$

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

$$4. \frac{8}{15} - \frac{1(3)}{5(3)} \leftarrow \text{need common denominator}$$

$$\frac{8}{15} - \frac{3}{15}$$

$$\frac{5 \div 5}{15 \div 5} = \frac{1}{3}$$

$$5. \frac{3}{4} - \left(-\frac{1}{2}\right) \frac{(2)}{(2)}$$

$$\frac{3}{4} + \frac{2}{4}$$

$$\frac{5}{4} = 1\frac{1}{4}$$

$$6. 8\frac{1(3)}{4(3)} - 2\frac{5(2)}{6(2)}$$

$$8\frac{3}{12} - 2\frac{10}{12}$$

$$7\frac{12}{12} + \frac{3}{12}$$

$$7\frac{15}{12} - 2\frac{10}{12}$$

$$5\frac{5}{12}$$

7. At the start of a trip, a car's gas tank contains 12 gallons of gasoline. During the trip, the car consumes $10\frac{1}{8}$ gallons of gasoline. How much gasoline is left in the tank?

$$12 - 10\frac{1}{8}$$

$$11\frac{8}{8} - 10\frac{1}{8}$$

$$1\frac{7}{8} \text{ gallons}$$

8. A fish was swimming $3\frac{1}{2}$ feet below the water's surface at 7:00 a.m. Four hours $\rightarrow 11:00 \text{ a.m.}$ later, the fish was at a depth that is $5\frac{1}{4}$ feet below where it was at 7:00 a.m. What rational number represents the position of the fish with respect to the water's surface at 11:00 a.m.?

$$-3\frac{1^{(2)}}{2^{(2)}} - 5\frac{1}{4}$$

$$-3\frac{2}{4} - 5\frac{1}{4}$$

$$-8\frac{3}{4} \text{ from the water's surface}$$

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Lesson 7 Practice

Addition and Subtraction of Rational Numbers

Find the following sums or differences in simplest form.

1. $\frac{5}{8} + \frac{6}{8} = \frac{11}{8}$

$1\frac{3}{8}$

2. $(-\frac{5}{14}) + \frac{7}{14}$ *ds1*

$\frac{2}{14} \div 2 = \frac{1}{7}$

3. $8\frac{2}{5} - 4\frac{4}{5}$

$7\frac{5}{5} + \frac{2}{5} = 7\frac{7}{5} - 4\frac{4}{5} = 3\frac{3}{5}$

4. $\frac{4(4)}{9(4)} - \frac{1(3)}{12(3)}$

$\frac{16}{36} - \frac{3}{36}$

$\frac{13}{36}$

5. $-\frac{2(4)}{3(4)} + \frac{3(3)}{4(3)}$

$-\frac{8}{12} + \frac{9}{12}$

$\frac{1}{12}$

6. $10\frac{3(2)}{5(2)} + 5\frac{1(5)}{2(5)}$

$10\frac{6}{10} + 5\frac{5}{10}$

$15\frac{11}{10} \rightarrow 16\frac{1}{10}$

7. A bird that was perched atop a $15\frac{1}{2}$ foot tree dives down six feet to a branch below. How far above the ground is the bird's new location?

$15\frac{1}{2} - 6$

$9\frac{1}{2} \text{ ft}$

8. Sonji and her friend Rachel are competing in a running race. When Sonji is $\frac{4}{10}$ miles from the finish line, she notices that her friend Rachel has fallen. If Sonji runs one tenth of a mile back to help her friend, how far will she be from the finish line?

$\frac{4}{10} - \frac{1}{10} = \frac{3}{10}$

$\frac{3}{10} = \frac{1}{2}$

NOTES: Lesson 15

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Multiplication and Division of Rational Numbers

RULES: Multiplying Fractions

1. Change mixed #'s to improper fractions
2. Cross reduce whenever possible
3. Multiply straight across numerators + denominators
4. Reduce / simplify

Find the following products in simplest form.

$$1. \frac{1}{2} \cdot \frac{4}{5} = \boxed{\frac{2}{5}}$$

$$2. -\frac{1}{7} \cdot \frac{3}{5} = \boxed{-\frac{3}{35}}$$

$$3. -\frac{4}{9} \cdot (-5) = \frac{20}{9} = \boxed{2\frac{2}{9}}$$

$$4. 2\frac{3}{5} \cdot \frac{1}{3}$$

$$\frac{13}{5} \cdot \frac{1}{3} = \boxed{\frac{13}{15}}$$

$$5. 2\frac{4}{15} \cdot (-6\frac{1}{4})$$

$$\frac{34}{15} \cdot (-\frac{25}{4})$$

$$-\frac{85}{6} = \boxed{-14\frac{1}{6}}$$

$$6. -5\frac{5}{6} \cdot (-3\frac{6}{7})$$

$$-\frac{35}{6} \cdot (-\frac{27}{7})$$

$$-\frac{45}{2} = \boxed{-22\frac{1}{2}}$$

7. A hiker averages $6\frac{3}{8}$ kilometers per hour. If he hikes for $5\frac{1}{3}$ hours, how many kilometers does he hike?

$$6\frac{3}{8} (5\frac{1}{3})$$

$$\frac{51}{8} \cdot \frac{16}{3}$$

$$\boxed{34 \text{ km}}$$

$$3 \overline{) 51} \begin{array}{r} 17 \\ 3 \\ 21 \end{array}$$

RULES: Dividing Fractions

1. make all #'s fractions
2. Keep, change to multiplication, flip 2nd fraction KCF
3. cross reduce if possible
4. Multiply numerators + denominators straight across
5. Reduce/Simplify

Find the following quotients in simplest form.

8. $\frac{6}{7} \div \frac{1}{7}$

$$\frac{6}{7} \times \frac{7}{1} = \frac{6}{1} = \boxed{6}$$

9. $5 \div \frac{7}{15}$

$$\frac{5}{1} \cdot \frac{15}{7} = \frac{75}{7} = 10\frac{5}{7}$$

10. $-\frac{4}{6} \div \frac{10}{24}$

$$-\frac{4}{6} \cdot \frac{24}{10} = -\frac{8}{5} = \boxed{-1\frac{3}{5}}$$

11. $\frac{3}{4} \div 5\frac{1}{2}$

$$\frac{3}{4} \div \frac{11}{2} = \frac{3}{4} \cdot \frac{2}{11} = \boxed{\frac{3}{22}}$$

12. $7\frac{1}{2} \div (-2\frac{5}{6})$

$$\frac{15}{2} \div -\frac{17}{6} = \frac{15}{2} \cdot -\frac{6}{17} = -\frac{45}{17} = \boxed{-2\frac{11}{17}}$$

13. $-3\frac{4}{9} \div (-2\frac{1}{3})$

$$-\frac{31}{9} \div -\frac{7}{3} = \frac{31}{9} \cdot \frac{3}{7} = \frac{31}{21} = \boxed{1\frac{10}{21}}$$

14. Mrs. Lau rolls out 33 inches of dough to make noodles. If the noodles are $\frac{3}{8}$ of an inch wide, how many noodles will she make?

$$33 \div \frac{3}{8} = \frac{33}{1} \cdot \frac{8}{3} = \boxed{88 \text{ noodles}}$$

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Lesson 15 Practice

Multiplication and Division of Rational Numbers

Find the following products in simplest form.

1. $\frac{1}{3} \cdot \frac{3}{5}$
 $\frac{1 \cdot 3}{3 \cdot 5} = \frac{1}{5}$

2. $-\frac{8}{13} \cdot \frac{26}{27}$
 $-\frac{8 \cdot 26}{13 \cdot 27} = -\frac{2}{3}$

3. $-5\frac{1}{4} \cdot (-12)$
 $-\frac{21}{4} \cdot -\frac{12}{1} = \frac{63}{1} = 63$

4. $4\frac{1}{2} \cdot \frac{2}{3}$
 $\frac{9}{2} \cdot \frac{2}{3} = \frac{3}{1} = 3$

5. $-1\frac{4}{9} \cdot 2\frac{4}{7}$
 $-\frac{13}{9} \cdot \frac{18}{7} = -\frac{26}{7} = -3\frac{5}{7}$

6. $-10\frac{2}{6} \cdot (-3\frac{3}{4})$
 $-\frac{31}{3} \cdot -\frac{15}{4} = \frac{355}{4} = 88\frac{3}{4}$

7. Anna wants to make 4 sets of curtains. Each set requires $5\frac{1}{8}$ yards of fabric. How much fabric does she need?

$5\frac{1}{8} (4)$
 $\frac{41}{8} \cdot \frac{4}{1} = \frac{41}{2} = 20\frac{1}{2} \text{ yds}$

8. One sixth of the students at a local college are seniors. The number of freshmen students is $2\frac{1}{2}$ times that amount. What fraction of the students are freshmen?

$\frac{1}{6} \cdot 2\frac{1}{2}$
 $\frac{1}{6} \cdot \frac{5}{2} = \frac{5}{12}$

Find the following quotients in simplest form.

9. $\frac{3}{8} \div \frac{3}{12}$

$$\frac{3}{8} \cdot \frac{12}{3} = \frac{36}{24} = \frac{3}{2} = 1\frac{1}{2}$$

10. $8 \div \frac{1}{3}$

$$8 \cdot \frac{3}{1} = 24$$

11. $\frac{12}{5} \div \left(-\frac{9}{10}\right)$

$$\frac{12}{5} \cdot -\frac{10}{9} = -\frac{120}{45} = -\frac{8}{3} = -2\frac{2}{3}$$

12. $\frac{3}{8} \div 6\frac{1}{4}$

$$\frac{3}{8} \div \frac{25}{4} = \frac{3}{8} \cdot \frac{4}{25} = \frac{12}{200} = \frac{3}{50}$$

13. $-2\frac{3}{5} \div 1\frac{1}{4}$

$$-\frac{13}{5} \div \frac{5}{4} = -\frac{13}{5} \cdot \frac{4}{5} = -\frac{52}{25} = -2\frac{2}{25}$$

14. $-8\frac{1}{3} \div \left(-2\frac{5}{9}\right)$

$$-\frac{25}{3} \div -\frac{23}{9} = \frac{25}{3} \cdot \frac{9}{23} = \frac{75}{23} = 3\frac{6}{23}$$

$$\frac{23}{69} \cdot \frac{75}{23} = \frac{75}{69} = \frac{25}{23}$$

15. Julia has $3\frac{1}{2}$ pounds of dog food. She plans to split it equally among her 7 dogs. How much dog food will each dog receive?

$$3\frac{1}{2} \div 7 = \frac{7}{2} \cdot \frac{1}{7} = \frac{1}{2} \text{ lb}$$

16. A batch of cookies requires $1\frac{1}{2}$ cups of sugar. How many batches can Ty make with $7\frac{1}{2}$ cups of sugar?

$$7\frac{1}{2} \div 1\frac{1}{2} = \frac{15}{2} \div \frac{3}{2} = \frac{15}{2} \cdot \frac{2}{3} = 5$$

$$5 \text{ batches}$$

NOTES: Lesson 16B
Properties of Numbers

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Vocabulary:

Property – a general rule that is used with a calculation.

Reciprocal - the multiplicative inverse (# flipped) $\frac{3}{4} \rightarrow \frac{4}{3}$ $5 \rightarrow \frac{1}{5}$

Addition Properties	Words	Examples
Commutative Property	<u>change order</u>	$2+3 = 3+2$
Associative Property	grouping changes	$2+(3+4) = (2+3)+4$
Identity Property	answer stays the same	$2+0 = 2$
Inverse Property	adding the opposite	$2+(-2) = 0$
Opposite of a sum is the sum of the opposites	multiply the negative sign through	$-(2+3) = -2 + -3$

Multiplication Properties	Words	Examples
Commutative Property	Change order	$2 \cdot 3 = 3 \cdot 2$
Associative Property	grouping changes	$2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$
Identity Property	answer stays the same	$2 \cdot 1 = 2$
Inverse Property	multiply the reciprocal	$2 \cdot \frac{1}{2} = 1$
Zero Property	anything times 0 is 0	$2 \cdot 0 = 0$

Distributive Property

Multiply # next to () by every thing inside

$$2(3+4) = 2(3) + 2(4)$$

Examples:

1) $2 \cdot (5 \cdot n) = (2 \cdot 5) \cdot n$ assoc. of $\cdot$

2) $42 + x + y = 42 + y + x$ Comm. of $+$

3) $3x + 0 = 3x$ additive identity

4) $(3 \cdot m) \cdot 2 = 2 \cdot (3 \cdot m)$ Comm of $\cdot$

5) $a + (b + 12) = (b + 12) + a$ Comm. of $+$

6) $(5 + x) + 0 = (5 + x)$ additive identity

7) $a \cdot b \cdot 1 = a \cdot b$ multiplicative identity

8) $-4 + 4 = 0$ additive inverse

9) $3 \cdot \frac{1}{3} = 1$ multiplicative inverse

10) $9(ab) = (9a)b$ assoc. of $\cdot$

11) $-(3 + 2) = -3 + (-2)$ Opposite of sum = sum of opposites (distributing)

12) $4 + (5 + 6) = (4 + 5) + 6$ assoc. of $+$

13) $-8\frac{5}{8} = -8 + (-\frac{5}{8})$ opposite of sum = sum of opposites

14) What value of a will make the equation a true statement? $a =$ $-\frac{7}{12}$

$$\left(-\frac{3(3)}{4(3)} + \frac{4(4)}{3(4)}\right) + a = 0$$

$$-\frac{9}{12} + \frac{16}{12}$$

$$\frac{7}{12} + a = 0$$

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Lesson 16B Practice

Properties of Operations

Name the property shown by each statement.

1. $9 \cdot 6 = 6 \cdot 9$

comm. prop of $\cdot$

2. $m + 0 = m$

additive identity

3. $14 \cdot 1 = 14$

mult. identity

4. $2 + (8 + 3) = (2 + 8) + 3$

assoc. of $+$

5. $x + y = y + x$

comm. of $+$

6. $(m + 2) + n = n + (m + 2)$

comm. of $+$

7. $6x + 0 = 6x$

~~add.~~ add. identity

8. $12(xy) = (12x)y$

assoc. of $\cdot$

9. $m \cdot n \cdot 1 = m \cdot n$

mult. identity

10. $8 + -8 = 0$

add. inverse

11. $-(7 + 10) = -7 + (-10)$

opposite of sum = sum of opposites

12. $\frac{1}{4} \cdot 4 = 1$

mult. invers.

13. $18 \cdot \frac{1}{18} = 1$

mult. inverse

14. $-9\frac{9}{10} = -9 + (-\frac{9}{10})$

opposite of sum = sum of opposites

15. What value of a will make the equation a true statement? $a = \underline{-\frac{1}{6}}$

$$\left(-\frac{2(2)5}{3(2)6}\right) + a = 0$$

$$-\frac{4}{6} + \frac{5}{6}$$

$$\frac{1}{6} + a = 0$$

Explain how you arrived at your solution. Found common denominator
added to get $\frac{1}{6}$ and the opposite of
 $\frac{1}{6}$ is $-\frac{1}{6}$.